Toward a Political Economy of Mathematics Education

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Why do schools teach the mathematics that they do? In this essay, Houman Harouni asserts that educational institutions offer mathematics standards and curricula without providing convincing justifications and that students are tested on content whose purpose neither they nor their teachers clearly understand. He proposes a theoretical framework for understanding the content and pedagogy of school mathematics as a set of practices reflecting sociopolitical values, particularly in relation to labor and citizenship. Beginning with a critical study of the history of mathematics instruction, Harouni traces the origins of modern math education to the early institutions in which mathematics served a clear utilitarian purpose, and in the process he unearths common, unexamined assumptions regarding the place and form of mathematics education in contemporary society.

The Why Questions of Mathematics

Nowhere in the discourse on math education do we come across a clear explanation for why schools teach the mathematics that they do. There is certainly plenty of literature on *how* and *what* to teach, but there seems to be a willingness to ignore the fundamental questions of what has arguably become the most problematic subject matter in schooling. The Common Core State Standards (2014), a set of content standards for American schools, for example, simply cites another study (National Research Council, 2009) as its justification for the importance of mathematics as a subject before moving on to make a vast array of recommendations on content and pedagogy. This latter study in turn passes the burden by citing other texts (e.g., National Research Council, 2001), which do not go beyond repeating the ready slogan that math is necessary for social and economic participation. Similar halls of mirrors are set up everywhere, special hiding places in which assumptions about mandated learning perpetuate themselves and produce glossy "new" curricula, ad infinitum.[1]

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The problem of unjustified school mathematics is not unique to the American and European contexts. In step with the globalization of mandatory schooling, it has reached near-universal status. Studies that purport to show the differences among various national contexts unwittingly achieve the opposite by highlighting the extreme similarity of national curricula (see English & Bartolini Bussi, 2008; Leung, Lopez-Real, & Graf, 2006; Wong, Hai, & Lee, 2004). The fact that international evaluations manage to apply themselves to almost every country is assumed to mean that there is a basic set of universal mathematical skills that should be tested. What it really indicates, however, is that there is a similar mathematics training that occurs around the world regardless of societal backgrounds. These similarities involve content, sequence of topics, and pedagogy, and they revolve around certain core ideas regarding what mathematics is and does.

Let's consider the first few months of an average first grader's math education. Her learning begins with counting—as opposed to, say, geometric reasoning or comparing quantities. She immediately learns to think of numbers as quantities of similar, individual objects (7 apples, 10 oranges), not measures or relationships. This type of reasoning is expressed and taught through a form of fairy tale—the word problem:

> Susan has 12 oranges. Her mother gives her 15 more. How many oranges does she have now?

The oranges have no identity of their own; once you dump them into a pile, you will not be able to tell one from the other. In the context of a word problem, they do not even have an actual group identity; we do not know where they come from or what Susan was doing with a dozen oranges in the first place, let alone why her mother should give her 15 more.

Before the child is able to count even up to 100, she is asked to perform arithmetic operations, and addition is universally the first operation she will learn. It will take months before her teacher introduces subtraction, and it will sometimes be a year or two before the child will look at multiplication and division. Until high school graduation, pen and paper will remain the dominant instruments of performing mathematics. Rulers, compasses, and protractors make their brief appearances but are quickly set aside in favor of abstract calculation problems. The world, most math textbooks tell us, is full of things that demand immediate manipulation of their numbers.

This model of elementary school math is as resilient as it is prevalent. The basic curriculum has remained largely unchanged since the founding of modern schooling (National Council of Teachers of Mathematics, 1970; Phillips, 2011). It is at the base of most national curricula. At the same time—and also from the very first days of public schooling—a debate has raged around math education. There have always been reformers to call the dominant approach outdated or unsound and to suggest different approaches. The result is a variety of alternative curricula that partially challenge the assumptions of dominant school mathematics. The Montessori model, for example, presents numbers as differences in magnitude rather than quantity (Montessori, 1914); Waldorf schools teach that all numbers are driven from and add up to a larger whole, an essential unity (Aeppli, 1986); the New Math curriculum based its definition of numbers on set theory (Hayden, 1981). The existence of these alternative models deepens the problem regarding the relationship between math and society: if (and this is only an unexamined assumption) math education fulfills a societal need, then an alternative approach indicates a different attitude toward labor and citizenship. What constitutes these differences? The persistence of the debate also indicates that the resilience of the traditional model is not due to lack of know-how. What, then, is the source of its resilience?

Academic answers to these questions are often either purely pedagogical, studying the efficacy of a method in teaching a certain topic, or idealistic, distinguishing curricula according to a measuring rod with poles such as problem solving versus rote learning. The disconnect between these explanations and societal issues eventually breeds a third, more radical explanation: perhaps school math has *nothing* to do with usefulness and is primarily a product of an education system in which the main purpose is not learning but becoming socialized and receiving certificates (e.g., Lave, 1992; Lundin, 2010b). The dominant format of math, then, is dominant either because it enjoys the strength of tradition or because there is something in math itself that inevitably leads to the current format. In this context, reform agendas appear as mechanisms through which schooling maintains itself by venting out and neutralizing all radical critiques of its form and content (Lundin, 2010a). Such arguments, despite their usefulness in creating a strong political stance against the status quo, are purely negative. They do not contain any element of redemption, even beyond schooling. Nor do they help us understand the current situation in any detail.

The first step toward a radical reformulation of math education is a genealogical understanding of current assumptions and practices. There is in this essay something of the resistant student's most common query to her math teacher: "Why are we learning this?" Along with those students who, in the face of relentless testing and intricate mechanisms of reward and punishment, continue to insist on their right to know, I want to suggest that the common answer "Because it's good for you!" is not good enough at all and should be treated with extreme suspicion. I propose a framework for understanding these questions—a historical framework wherein we can begin to place ourselves before the phenomenon of contemporary math education.

Addition Is Not Addition: Three Approaches

Let's begin where schools begin, with Susan's 12 oranges. The question of how many oranges she would have if her mother gave her 15 more is represented as follows:

12 + 15 = ?

This format is by far the dominant way of teaching addition in classrooms. It is so familiar as to seem like the basic building block of mathematical thinking rooted in the most basic social interactions. But what if we imagine something else, which is just as basic:

27 = ?

There are fundamental differences between these two questions. The first is interested in a particular task; it demands to know what will happen if two numbers are added together. The equal sign in this problem is more or less a command to compute.[2] Coincidentally, this is also the calculator's understanding of the equal sign. By contrast, in the second, the equal sign is asking a question regarding the *meaning* of 27. In the first problem, there can only be one answer. For 12 + 15 to become anything other than 27, the world must come apart at the seams. In the second format, however, the answers are innumerable, and they increase as one's knowledge of arithmetic expands.

Another alternative, based on real-world interactions, challenges the faith one places in either of the above formats. We might ask, "12 of what?" or "15 of what?" What is the result of adding 12 oranges to 15 apples, for example? Adding 12 meters of rope to 15 square centimeters of wood will not give 27 of any unit. If we imagine numbers as referring to real objects in a real context, then the addition sign would rarely (only in very special circumstances) imply a simple accumulation of quantities; it would refer, instead, to a much more complicated process that can only be understood in its proper setting. From this perspective, the equal sign is neither a command to compute nor a question of meaning; it symbolizes an aspect of work performed on materials.

I am not merely trying to suggest alternative ways of teaching addition but, rather, that there is a qualitative difference among these three formats. Functionally, I have described the difference by suggesting how each can teach a different attitude toward mathematics, but what I have not described are the contexts that give shape to each approach and the reasons why the first format has come to dominate the teaching of arithmetic everywhere. Such a description is a necessary step in order to free our thinking from its ahistorical foundations.

Three Historical Venues: Shop Floor, Grammar Schools, and Reckoning Schools

By the sixteenth century in Western Europe, mathematical learning was taking place in three very different institutional settings.[3] The first, and the one least covered in the history of education, was the institution of apprenticeship. In this setting, craftsmen learned their trade through direct contact and on-the-job training with a master and other apprentices. Many crafts involved what can loosely be thought of as mathematical skills: masons and carpenters, shipbuilders and wheelwrights each had the need for a set of numerical or geometrical systems or maneuvers. However, it would be a mistake to think of these mathematical practices as separate from the actual work of the workshops. The carpenter's act of measuring planks, for example, might involve operations similar to what one learns in school today, but the artisan's math is intertwined with the materials and instruments of his work (Smith, 2004). The ruler he uses defines the meaning of numbers for him. A plank of oak serves a different function and can bear a different weight than a plank of walnut, even if the measurements are precisely the same.

Within the institution of apprenticeship, the characteristics of performed labor defined the mode and content of the artisan's mathematical training. Education for artisans did not mean storing up knowledge to use at a later time: learning rose out of performing or observing a useful function. The little we do know about what happened on European shop floors tells us that the apprentice, after a year or so of observing the master and performing simple manual labor, took up a simple commission that, once complete, was sold and used (De Munck, 2007). The products of his labor predefined all his actions, including his learning.

The second place for learning math in sixteenth-century Europe was the grammar school.[4] These constitute a gray area in our account, because until the late sixteenth century, most grammar schools did not teach mathematics (Howson, 1982; Struik, 1936). Their primary task, as their name implies, was the teaching of classical languages; even local languages were, for centuries, a secondary concern and only gained attention as Latin and Greek lost their prominence (Thompson, 1960). Grammar schools served to teach "culture" to the sons (and sometimes daughters) of educated commoners—physicians, pastors, lawyers, and town officials. The occasional mathematics taught in grammar schools was closely tied to that taught in the universities of the time, which corresponded to a knowledge of the classics, and included some mathematics (Howson, 1982; National Council of Teachers of Mathematics, 1970). By the age of thirteen or fourteen, students were done with their grammar school education. Those for whom the basics sufficed joined their families or a master for professional training as apothecaries, copyists, notaries, scrivners, and stationers. Those headed for ecclesiastical or legal careers entered the university.

As a rule, university math shirked any emphasis on calculation and instead focused on the relationship between numbers in their "pure" form (i.e., more "27 = ?" than "12 + 15 = ?"). The textbooks began with an introduction to Arabic numerals that spanned no more than two or three pages and then moved on to brief definitions of various types of numbers (e.g., odd, even, prime). Geometry, which was heavily Euclidean, took up the bulk of students' learning in mathematics. Arithmetic texts written for use in both universities and grammar schools treated the subject quite theoretically. They emphasized definitions rather than application, rarely contained sample problems from daily life, and concentrated on the logical and intuitive relationships among numbers (Jackson, 1906).

However, neither of the models—the one used in universities/grammar schools and the one taught on shop floors—resembles the dominant modern form of teaching elementary mathematics. In these two models, there is no intensive teaching of arithmetic operations, nor any practical word problems about giving and taking, buying and selling, trading and borrowing. Euclidean geometry barely prepares one for calculating the area and perimeter of shapes, which is the main focus of elementary school geometry. We do find these familiar elements in the third venue for teaching mathematics. It may be unfamiliar by name but is easily recognizable by curriculum. In England it was referred to as a "reckoning school," and its teacher was a reckonmaster (in Italy a *maestro d'abaco,* in France a *maistre d'algorisme,* and in Germanic territories a *Rechenmeister*).

Reckoning schools first appeared in the fourteenth century in the commercial cities of Italy and later spread along the routes of the Hanseatic League trading confederation (Swetz & Smith, 1987). In 1338 Florence, the most important center for teaching mercantile mathematics, there were 6 reckoning schools; by 1613, with the rise of the mercantile economy in Europe, Nuremberg alone boasted some 48 reckonmasters (Swetz & Smith, 1987), and Antwerp was home to 51 (Meskens, 1996). Students in reckoning schools were the children of merchants and accountants sent at about the age of 11 or 12 to study commercial arithmetic with a reckonmaster (Jackson, 1906; Swetz & Smith, 1987). As an example, in Florence in 1519, Francesco Galigai taught a course of instruction for boys between ages 11 and 15. The course lasted about two years, and classes met six days a week. His curriculum, typical of its kind (see Goldthwaite, 1972; Howson, 1982; Swetz & Smith, 1987), contained seven consecutive sections, each paid for separately:

1. Addition, subtraction, and multiplication (including memorization of algorithms and fact tables)
2. Division by a single digit
3. Division by a two-digit number
4. Division by a three-digit number or more
5. Fractions (basic operations, used in problem situations)
6. Rule of three
7. Principles of the Florentine monetary system

Textbooks on commercial arithmetic from that time went beyond Galigai's basic curriculum and dedicated larger sections to monetary systems, topics that include rates of interest, partnership in trade, and currency exchange (Harouni, 2013; Jackson, 1906). The only geometry that made its way into these texts and classrooms was land surveying, the calculation of areas and perimeters (Harouni, 2013).

The similarities between reckoning school content and most modern curricula are striking—the same emphasis on calculation, the same sequence of operations, the same computational view of geometry. Reckoning textbooks share even more features with their modern counterparts in schools (see Jackson, 1906). Without exception, they emphasized algorithms for solving operations and demanded a memorization of the most salient arithmetic facts. Salience was a factor of the frequency with which a set of numbers appeared in trade (e.g., in England, 12 was an important number, since there were 12 pence to each English shilling). There was little or no attempt to present the underlying principles that make an algorithm work. For example, the author of *Treviso Arithmetic* (ca. 1478), the earliest printed arithmetic book available, introduced his readers to two-digit addition:

> We always begin to add with the lowest order, which is of least value. Therefore, if we wish to add 38 to 59 we write the numbers thus:
>
> 59
> 38
> Sum: 97
>
> We then say, "8 and 9 make 17," writing 7 in the column which was added, and carrying the 1 . . . This 1 we now add to 3, making 4, and this to 5, making 9, which is written in the column from which it is derived. The two together make 97.

This approach—teaching an algorithm without concern for its mechanism—holds for the vast majority of reckoning books (Jackson, 1906). Alongside the algorithms, the texts also feature a large number of word problems for each topic; many of them, mutatis mutandis, could have been written yesterday.

While in reckoning schools we see many elements of contemporary "traditional" math education, in certain alternative approaches we observe stronger similarities to the math taught in the other sixteenth-century venues. Waldorf and New Math, with their emphasis on teaching the nature of numerical rather than calculative relationships, for example, display something of university mathematics; Montessori's reliance on math arising from physical experience with objects is closer to the craftsman's approach. Reckoning school math, however, is by far the one most closely and widely replicated in the contemporary context. Perhaps the major difference between the reckoning program and the vast majority of current curricula is that the more complex monetary practices—dividing profits in a partnership, calculating complex loans and inheritances, for example—have not made it into our classrooms.

The Role of History

There are two distinct ways for drawing connections between sixteenth-century institutions and contemporary math education—the chronological approach and the theoretical approach. Proceeding chronologically, we draw lines that show the development of modern elementary school from its antecedents; moving step by step, we try to establish whether one practice led to another, whether one institution influenced the next, until we arrive at the present. The similarities between reckoning and modern school math are not arbitrary.

By the late seventeenth century, the merchant classes had gained enough power to impact the grammar and parochial schools' curricula (Howson, 1982; Swetz & Smith, 1987). This was partly a matter of finances: accountants and merchants did not want to pay for two types of education—for literacy gained in grammar schools and for numeracy gained from reckonmasters—and so chose to combine the two for a single fee (Jackson, 1906). More importantly, the other educated and well-to-do city dwellers also found themselves in closer contact with commerce, bookkeeping, debt, and other activities that are part and parcel of middle-class life in a money economy. Until the rise of the mercantile and capitalist economy in Europe, the higher classes attached a strong stigma to arithmetic due to its connection with trade and banking. Almost all sixteenth-century textbooks began with a sort of apologia, the author making a case for the intellectual and spiritual value of arithmetic (Davis, 1960). However, by the late seventeenth century, the stigma attached to counting and accounting departed on the same wave that wrested the control of social life away from the church and placed it in the hands of the bourgeoisie. The demand for commercial arithmetic had turned the subject into a main feature of middle-class education in most of Western Europe.

Within a few generations, a new type of schoolteacher emerged out of this process, a compromised combination of the old grammar school teachers and reckonmasters: someone able to teach basic reckoning but not versed in more advanced financial applications of mathematics. Since grammar schools still had to prepare children for the university, their teachers were also charged with teaching a smattering of the type of mathematics practiced in higher education. Some, usually rote, learning of Euclidean geometry became the most salient way of fulfilling this requirement. In upper-class schools, where education was meant to equip students with knowledge of higher culture, the philosophical approach to math remained more prominent than it did in poor or working-class schools.

Grammar school (which in certain English-speaking areas is still the name used for elementary education) in many ways formed the foundation of public education as we know it today. Its structure and values were carried over to the schools of the poor, the peasants, and the working class by teachers and reformers who were themselves products of grammar schooling. Across various eras, society has experimented with slight modifications in the grammar school framework, hoping to make it better fit various contexts: the factorylike Lancasterian system for the poor and working class and the college prep model that shaped some of the private schools for the rich, to name two (Bowles & Gintis, 1977; Howson, 1982). By and large, however, the grammar school—carrying the reckoning school implant—was the blueprint for the schools established by the state for the education of the public. But as it expanded beyond its original base, the grammar school model became involved in a strange dialectic: it imposed itself on classes and entire cultures whose interests it did not represent. One result of this institutional mismatch may have been the elimination of more complex financial mathematics from curricula meant for working- and lower-middle-class students.

And so, in the step fashion, history brings us to the present. This chronological account is an efficient way of presenting the sources of contemporary practice. It seems to provide us with a twofold task: to identify and replace outdated practices and to make education more relevant to marginalized classes and cultures. Here, however, we face an essential problem: the historical forces that keep the old practices in place or that make one group marginal to another are still present in society. By confining history to the past, viewing it as merely a source of habits, we stop thinking historically. In a purely chronological analysis, the resilience of practices that seem useless or ritualistic is attributed to institutional foot-dragging—if only we could reorganize schools according to different values, then things would fix themselves. This is the perspective underlying any reform agenda that chooses to refer to its target primarily as "traditional" schooling. Yet, where will those new values come from, given that the very lenses through which we judge the world are historically formed? Even lethargy rests on something more than mere laziness and ignorance.

At the beginning of this essay I proposed that we explore the history of mathematics education in order to understand why it has taken on its present form. However, to understand an existing practice, or to create a new one, it is not enough simply to trace its genealogy. History is most powerful precisely where it cannot be traced, where it seems to spin out entirely new practices, or where it adopts an old practice in a very different context, thus rendering it new. To grasp the interplay of historical forces, stepwise historicism is deficient. To analyze our own lenses, we need the incisive blade of theory. In the case of mathematics education, we are dealing with an overwhelming economic fact that we hold in common with sixteenth-century Europe: commercial and administrative calculation is *still* the dominant intellectual activity of our societies. We are not merely inheritors of reckoning. We *are* reckoners—and perhaps academics, artisans, politicians, and so on—and the math we teach contains our attitudes. This is the starting point for a theoretical understanding.

Categories of Mathematics

Commercial-Administrative Mathematics

We can expand what we know about reckoning to think of it as a category of mathematics. I refer to this category as *reckoning* or *commercial-administrative mathematics*. To speak of a category, however, is primarily to set up a device for thinking. In actual, historical conditions, the type of math that emerges from commercial-administrative practice is shaped in relationship to the larger social and institutional settings. Nonetheless, there remain enough similarities across a wide variety of contexts having to do with the similar place of the merchant and administrator in relation to economic production that it becomes useful to cluster these similarities under a meaningful title.

Let us hold reckoning—that is, extract it from the rapid flood of history—for just a moment, knowing that it will immediately slip out of our hands and back into the current. Because counting and exchange appear as fundamental human activities, there is a tendency to think that commercial-administrative math is also simply an outgrowth of human nature. For example, Kamii (1982) draws on Piaget, saying, "Every culture that builds any mathematics at all ends up building exactly the same mathematics, as this is a system of relationships in which absolutely nothing is arbitrary." Both Piaget and Kamii ignore the premise that for math to turn into what they describe, there first has to be a need to look for and construct relationships in which "absolutely nothing is arbitrary." The underlying assumptions of such conclusions are false. Counting is not a "natural" human activity, as hunter-gatherer tribes that develop only a "one-two-many" counting system have clearly demonstrated (Gordon, 2004; Pinker, 2007). Moreover, the movement from counting sheep to reducing human labor into abstract quantities and keeping books on everything (shipments of olive oil from Greece, the number of workers building a dam, an entire nation's taxes) is not merely a matter of expanding on the basic principles of counting. The development—its form and degree of proliferation—is neither natural nor inevitable but instead cultural.

The history of commercial-administrative math moves alongside the history of accumulated labor power. Without the opportunity for products and work hours to accumulate, there would be nothing to bookkeep or trade at a level that would require a special discipline of mathematics. This overruling mindset impacts every aspect of reckoning mathematics. First and foremost, in reckoning, number is a placeholder, referring to values drawn from the real world. In other words, number is never fully abstract or freestanding; instead, it is, at the last instance, the product of counting things. There is a special character to this counting that makes it different from, say, scientific or technical measurement. It is important to notice that if numbers are closely tied to counting, then it becomes quite problematic to apply them to more abstract concepts. As Russell (1919) put it, you cannot develop the various definitions of infinity from counting, because we could not possibly count an infinite number of things. Nor can certain fractions or irrational numbers be expressed in terms of counting. From a scientific perspective, then, commercial-administrative math—which Hoyrup (1994), when describing the mathematics of Babylonian scribes and the similar practices of reckoning schools in Europe, classifies as a form of "subscientific" mathematics—should have at some point in history died off, being unable to explore all the mathematical problems it engenders. In reality, due to reckoning mathematics' economic significance, it continues to dominate the lay perception of mathematics.

The second essential aspect of commercial-administrative math is that it treats calculations as ultimately representing predictable social interactions. The extensive use of word problems in reckoning textbooks is both a result and a promoter of this approach. Along with practical considerations, this approach gives rise to a host of psychological decisions that are particularly bold when expressed in pedagogy. There is, for example, no mathematical or practical reason for the unquestioned primacy of addition in reckoning school (and contemporary) education. There is no developmental reason that bars teaching subtraction first, since even young children can learn it at the same time as they learn addition (Starkey & Gelman, 1982). This suggests, rather, the commercial-administrative attitude is at work here: a certain degree of *accumulation* of resources is necessary before any of the other practical functions served by commerce or administration can take place. It is this sociological need for accumulation—seen as the source of all interactions—that can account for the seminal place of addition. The order of teaching operations in Europe prior to the rise of a money economy, for instance, was not always the same as it is today. For instance, Abraham ibn Ezra (ca. 1140) and Fibonacci (ca. 1202) began their lessons with multiplication and then went on to division, addition, and subtraction (Swetz & Smith, 1987). Today, Waldorf schools, which explicitly reject a calculative approach to elementary mathematics (see Aeppli, 1986, pp. 41–57), teach all four basic operations simultaneously.

The earliest appearance of commercial-administrative math in historical record is in clay tablets surviving from ancient Sumer and Babylonia (Friberg, 1999). A very large number of tablets has been recovered from the sites of scribal schools and allows a view of how scribes taught and learned the mathematics of their trade. What is astonishing is the similarity between many of these ancient methods and what we observe in reckoning texts or contemporary curricula: the same reliance on ready-made algorithms, the same use of word problems, the same tendency to reduce concrete things to a numerical value.

Scribes originally functioned as administrators for the various organs of society—the temple, the state, or private persons, depending on the era. They kept records on harvests, treasuries, and building projects. They calculated the amount of clay, straw, and bricks needed for each building and converted these into the labor and power needed to produce them (Friberg, 1996, 2007). One Babylonian example in particular anticipates our own math problems:

> If a man carried 420 bricks for 180 meters, I would give him 10 liters of barley. Supposing he finished after carrying 300 bricks, how much would I give him? (Friberg, 1996)

The modern versions usually involve a number of workers finishing a job in a certain number of days and the textbook asking us to calculate how long it would take a different number of workers to finish the same job. The Sumerian version is slightly more complicated, because it does not shy away from the calculation of wages. There is no confusion about why we are looking at work-hours—we want to know how much we have to pay.

How can bricks and distances, barley and men, such disparate things, all interact so seamlessly with each other in a problem? By becoming reducible. Each entity is reduced to the labor power that it demands or embodies. It is hard to think of activities outside of commerce or administration that would dissolve the material identity of objects and people so readily. This reductionist tendency is so strong that it seeped into every aspect of the scribes' teaching, and eventually they arrived at nonsensical practice questions that had the student add up ants, birds, barley, and people, all into a pile. The tablet featuring the problem can be represented as follows (Friberg, 2005, p. 5):

649,539	barley-corns
72,171	ears of barley
8,091	ants
891	birds
(+) 99	people
730,791	

The tablet, in all likelihood, indicates a type of word problem, examples of which reappear in ancient Egypt and Europe and up to the modern era (Friberg, 2005). Notice how blithely the student is asked to add birds, ants, barley, and people. It was necessary for the young learner to get used to seeing things as interacting in this way. What does it mean to add ants to birds and to people? The teacher is unconcerned, because in his professional sphere, numbers present an interstitial virtual space within which all objects and people lose their material identity and blend into abstract value. As long as the student knows how to perform complex operations, the material aspect of numbers is unimportant. The majority of numbers are bereft of all meaning, except for their quantity as items and their value in exchange—and these two are easily convertible to one another. In modern times this exchange value is reified within the concept of money, to which all commodities are converted.

Commercial-administrative math always stands *beside* the process of production, turning objects and labor into abstract quantities. The reckoner is interested in the interaction between numbers because he needs to predict the outcome of contracts, exchanges, partnerships, or investments, the specifics varying according to the mode of production. In any setting he looks to laws of exchange or administration set in the past and calculates for the future—How long *will* it take the workers to finish the job? How much *will* we have to pay or feed them? What *will* the interest on this loan be? Labor power is time is product is remuneration. All these things are ultimately interchangeable. The future is a reliable one—the flowering of the seed of the owner's hope in an investment or of the administrator's promise to safeguard one. Without reliability, investment and exchange cannot proliferate. This is one reason why the types of problems reckoning mathematics uses for training are single-answer questions in which only one type of transaction can take place. The obsession with this final answer demands more and more efficient mechanisms of arriving at the answer. How these mechanisms work is not nearly as important as the assurance that they *do* work. Furthermore, the units that the word problems refer to are not nearly as important as their ability to hint at predetermined, simple interactions involving reducing people and objects to some value.

Ultimately, in modern curricula, when a textbook question talks about apples and oranges, it does not mean apples and oranges. It means money. And yet neither teacher nor student is aware of this underlying meaning. The actual mercantile and administrative content of math has all but disappeared from modern curricula, replaced by odd and nearly always spurious references to domestic activity (see Dowling, 1998; Lave, 1992). The general form, however, has remained the same. In this diffuse form, the commercial-administrative mind-set addresses itself to all things without ever revealing its own nature.

Artisanal Mathematics

While commercial-administrative math stands outside the process of creative labor, a different kind of mathematics emerges from *within* that process. It is shaped by the interactions among people, instruments, and materials. In creative labor, the material identity of things is not obliterated but transmuted. The mathematics of the artisan's workshop is therefore part of this transformation process—the workman's skill meeting the material. I refer to this type of activity as *artisanal mathematics*, keeping in mind that it is so different from other forms of math that often it does not even make sense to refer to it as "mathematics."

It is measuring, not counting, that constitutes the artisan's primary encounter with numbers. It is tempting to think of measurement as the counting of units, but this temptation is the result of our own early education. Teachers introduce students to measurement by handing them a ruler and asking them to measure the length of a line, counting the centimeters from zero. Actual measurement, however, is rarely so one dimensional, because in the real world it is rare for a craftsman to care about only one dimension of an object. Every piece of wood has a type, an age, a weight, a density, a hardness, and a minimum of three dimensions, and a carpenter takes into account a combination of these aspects in every stage of his work. Furthermore, the measure of something does not reveal itself primarily as a sum of units but as a comparison among objects that is then expressed in terms of units. So, to divide the length of something into equal parts, the carpenter does not need to use a measuring tape; he can hold a piece of string to the object, separate a section that equals the length, and then divide the string as many times as needed.

Embedded in these tensions is the fundamental difference between the meaning of units in commercial-administrative and artisanal mathematics. All units of measurement, including currency, are arbitrary social constructs. But whereas for the artisan measures represent aspects of the materials he works with, for the merchant all materials, including their measures, are aspects of the value he invests or earns. The artisan, unlike the merchant, can improvise useful units for his work on the fly—using a piece of string to express a length. The only time he needs to think in terms of conventional units (inches, grams, etc.) is when he is communicating with strangers.

The artisanal, or apprenticeship, model of learning—watching an experienced worker closely, emulating complex skills that take into account many aspects of the work at the same time, recreating artifacts, assessing projects—is a necessary extension of the complex relationship between people and materials. Therefore, textbooks, worksheets, word problems, and explanations that, if not engaging, can at least communicate the basic ideas of reckoning are completely inadequate for the learning that creative labor requires. They cannot convey the complexity of things as needed. Even merchants who do more than basic accounting are partly trained on the job.

Artisanal learning did not help shape public schooling because of the rise of industrial capitalism, which rapidly eroded the influence of the artisan class. By the late seventeenth century in Europe, apprenticeships were proving expensive and outdated (De Munck, 2007). Technological progress limited the skills needed by the majority of workers on the shop floor, and increasingly the combination of simple wage labor and machinery replaced the work of trained artisans. Today, aspects of the apprenticeship mode of learning survive in engineering, art, and technical schools. In places where, under the influence of other academic subjects, training moves away from dealing directly with tools and materials, the student has to learn the real skills of her trade *on the job,* working with specific tools under the supervision of more experienced workers. The bond between the content of artisanal work and the pedagogy used to teach it remains strong: one demands the other.

In modern elementary education, we rarely come across any instances of artisanal mathematics. Therefore, it is harder to imagine how a learning model based on an artisanal attitude would differ from standard classroom mathematics. We do find at least one well-documented attempt in the work of Maria Montessori, who came upon an artisanal approach to teaching in a roundabout way—not through an attempt at training technicians but through close observation of very young children interacting with their environment. Early on she discovered that her kindergarten students were happiest and most focused when working independently with materials. She concluded that curriculum and content should become embedded in tangible things, "didactic materials," that embody certain relationships for the child to explore (Lillard, 2005; Montessori, 1914). This emphasis on materials brought Montessori to certain key elements of artisanal learning. The complexity of the materials meant that children learned better when watching someone else perform a task than when listening to explanations or copying down problems. Montessori decided that for her mode of learning to be effective, the room had to be full of people doing work. There was no way for all this work to be led by the teacher. Therefore, she conceived of classrooms that housed children of various ages, just as shop floors held workers of varying abilities.

However, this artisanal attitude, strong in the Montessori pedagogy for the early grades, gives way under the pressure of the common notion of mathematics for the simple reason that there is no real production process in Montessori schools. By the middle of the elementary years, Montessori materials turn into means of expressing relationships that are not creative but calculative (e.g., blocks that are expected to represent quadratic equations). So, ultimately, the absence of a strong theory for the purpose of math eventually brings the Montessori approach back to the dominant model.[5]

Philosophical Mathematics

In studying sixteenth-century Europe we can conceive of a third type of mathematics, the one corresponding to the math practiced in universities. This type of math stands neither inside nor beside productive labor. Its product is neither an object nor an interaction in the world but an order in the mind. However, we should resist the temptation to brand this type of math as abstract and impractical, as this math, too, only reaches its development once it becomes part of a larger social practice. We can think of it as *philosophical mathematics*, using *philosophy* as a blanket term to cover also priestly and academic activities. It is exemplified in the work of Euclid, in the astronomical and astrological discourses of Muslim scholars, and in the discipline of pure math that is the practice of academic mathematicians. As in the other categories, a large number of different practices coalesce, their differences not merely practical but also cultural. Nonetheless, some important similarities remain.

Philosophical mathematics loves patterns. It draws them out because they hint at meaning, and meaning is the priest's and philosopher's sustenance. When philosophical math turns to numbers, it turns to their meaning, the patterns, and the logical connections they contain. Sixteenth-century books of arithmetic that were meant for grammar schools first discuss the common number patterns (odds, evens, naturals, etc.) before arriving at operations.

Books written prior to the dominance of money economics and commercial arithmetic often introduce all four operations at the same time (see al Bīrūnī, 1984; Isidore & Hall, 2006). What implicitly steals the show in every page of philosophical arithmetic (as in the question "27 = ?") is the concept of equality, the meaning of a number. A number in philosophical math is not so much the carrier of value or a signifier of relative quantity as it is a repository of self-referential relationships.

In philosophy or theology, rigor is not defined by how well a student predicts the outcome of a practical situation. Scholarly arithmetic texts of the sixteenth century generally contained abstract or "impractical" puzzles rather than word problems. Even when a word problem did involve the use of money, it had nothing to do with a business situation; such problems were not interested in predicting the result of an interaction and did not try to practice the student's hand at a specific, existing algorithm. Note, for example, this problem from circa 1540:

> Three men together have a certain amount of silver, but each one is ignorant of the amount he has. The first and second together have 50 coins, the second and third, 70 coins, the third and first, 60. It is required to know how much each one has. (Jackson, 1906).

There is nothing businesslike in this problem, despite its reference to money and merchants. What the student performs here will not help a merchant conduct his work, and it is definitely of no help to the three men in the problem.

The more we look at philosophical mathematics, the more we understand the regular complaint of academic mathematicians (e.g., Lockhart, 2009) that the subject taught in elementary and secondary classrooms is far removed from the mathematics that they know. But what they see as the discipline of mathematics is also far removed from many other perspectives. This disconnect is evident in the debate that surrounded the New Math curriculum in the United States. In the 1960s mathematicians designed a curriculum that updated math education and corresponded to contemporary "mathematical" practices, which meant neither engineering nor modern money mechanics but pure academic math (Phillips, 2011). The result was a curriculum that seemed even more removed from everyday life than the traditional curriculum. For this reason and others, its days in American elementary schools were numbered—it did not last through the 80's. It could not withstand the most simple of all attacks: kids raised on New Math were not quickly proficient in (reckoning) calculations. "Johnny can't count!" was the battle cry of the "back-to-basics" movement (Hayden, 1981).

New Math, with its emphasis on patterns, classifications, and analysis, is not the only philosophical approach to math education. The Waldorf curriculum, for example, relies on a spiritualist agenda with emphasis placed on all numbers adding up to one, thus promoting the idea of universal oneness. In fact, as far as modern philosophical developments are concerned, the dry, academic curriculum of New Math, corresponding to the academic worldview of its founders, was reactionary; its philosophical core was bolstered by willful ignorance of all radical philosophy, from Marx to Nietzche. In other words, nearly a century after radical philosophy showed that logic is formed in relation to society and human psychology, New Math attempted to teach mathematical logic that was nearly empty of any sociological and psychological connections.

The production of contemporary math curriculum represents the outcome of a particular dialectical battle between philosophical mathematics and commercial-administrative mathematics. In a strong money economy, as soon as philosophical mathematics leaves its specialized cloisters and addresses itself to the general public, it is fated to meet commercial-administrative mathematics. While commerce and administration, which rely heavily on mathematics, want philosophical math to submit to and reinforce their agenda, the philosophical mathematician wants to reassert her independent identity and in that attempt brandishes her stronger, more scientific version of mathematics. This dynamic brings about a synthesis that can take a variety of forms depending on the battleground. In public education, the result tends to be disappointing to proponents both of reckoning and of philosophical math, as they do not intend to change their own practice or view of the purpose of math but only the way in which the future generation is trained in it.

Alternatively, turning to Plato provides insights on a different means to come to a resolution on keeping commercial-administrative mathematics and philosophical mathematics distinct and unadulterated. When Socrates, in Book 7 of *The Republic*, browbeats Glaucon into accepting that arithmetic is an essential subject for training the rulers of a utopian city, he immediately has to qualify his statement by separating the two forms of mathematical practice:

> Then it would be appropriate, Glaucon, to prescribe this subject in our legislation and to persuade those who are going to take part in what is most important in the city to go in for calculation and take it up, *not as laymen do*, but staying with it until they reach the point at which *they see the nature of the numbers by means of understanding itself*, not like tradesmen and retailers, caring about it for the sake of *buying and selling*, but for the sake of war and for ease in *turning the soul* itself around from becoming to *truth and being* [all emphases added]. (Plato, 2004, p. 220)

Thus, Plato, never shy about his disdain for working and trading classes, attempts to prevent the above-mentioned dialectic by clearly delineating a space for each type of math: philosophical math for aristocrats, reckoning (and possibly artisanal) math for lay people.

That elementary school mathematics has managed to retain its basic shape for so long, unaffected by all scientific and philosophical developments, has partly depended on a similar solution. "Basic" math in public schools in the United States and many other nations is almost exclusively a watered-down commercial-administrative approach, while the more philosophical approaches are reserved for more advanced students or alternative, private institutions. The now-commonplace observation that algebra seems to act as an intellectual barrier against many, particularly working-class, students entering the more advanced topics in mathematics (see Moses & Cobb, 2001) unwittingly hints at the discrepancies between the two types of math. Historically, algebra emerged in the Middle East as a reordering of the mathematical practices of merchants (which possibly relied on earlier methods used by land surveyors) placed on scientific footing by philosophical mathematicians (Hoyrup, 1987). It contains the conflict.

Social-Analytical Mathematics

For the purpose of understanding current trends in math education, we must consider one more category of mathematics, one that forms in the meeting of philosophical and commercial-administrative practices. I refer to this category as *social-analytical mathematics.* Exemplified in the disciplines of economics and social statistics, this category can only arise once commercial-administrative math is already highly developed, in widespread use, and subject to analysis by competing groups. The earliest, extremely rudimentary recording of this type of practice comes from certain Greek city-states in the fourth century BCE, where state accounts were posted in public for scrutiny (Cuomo, 2001). The accounts themselves were products of commercial-administrative mathematics; their public posting and the way they were read by their intended audience, however, were the result of a social setting in which one accumulator of resources found himself accountable to another section of society. The most obvious form of such accountability—taxes—initiated a primitive form of social mathematics when the ruling class tried to analyze the population to know the safe margin of taxation or control.

In its advanced form, social-analytical mathematics requires a situation where competing interests can view and analyze the data *at the same time*, and this effort takes the form of an argument that requires new tools for reasoning and representation. Thus, economics and social statistics, which enable such scrutiny, grew into disciplines against the background of the class struggles that shaped nineteenth-century Europe. The scientific basis for both had long existed, sometimes for centuries (e.g., ninth-century mathematician al-Kindi used frequency analysis to decipher encrypted messages), but there had been no reason to use them as tools of social analysis.[6]

Thus, social-analytical mathematics contains a special tension. It can be used as an administrative tool either by further reducing people to abstract units in order to predict their behavior or by providing justification for the status quo. At the same time, it can work against administration, acting as a tool of critique that helps give definition to phenomena that previously appeared too diffuse and scattered to have clear meaning.

In employing social-analytical mathematics, Marxist and socialist economics break with prior forms of mathematical practice, turning all disciplines of commercial-administrative mathematics on their heads by placing workers in the role of the analyzer. In this intellectual tradition, math no longer serves only to convert labor power and nature into exchange value. Instead, it is used to inquire after the life of the worker, his individual or group interest in all commodities and interactions. Unlike a reckoning mind-set, where significance is defined in exchange value, here the individual using mathematics can refuse such a reduction. Where such a reduction is encountered, one tries to subvert the process by arriving instead at the human beings who created the value in the first place. For wage workers, this type of math can be said to be about citizenship because it concerns the asymmetric interests that define life in the marketplace.

Drawing on this promise of social-analytical mathematics, many progressive educators have argued for math education employing statistics and analysis to help students "read the world," as Frankenstein (1983) has put it. Frankenstein grouped these efforts under a global movement called "critical math education," which has resulted in a body of curricula that problematize the data on racial, economical, and gender inequalities, among other social problems. In a 2009 essay on word problems, Frankenstein offers the clearest articulation of a social-analytical approach to math education to date. In her approach, every mathematical product is to be seen as a codified social interaction meant for critical analysis. Numbers are to be used by the teacher to help describe the world and to also showing how numerical descriptions distort or hide reality. The purpose of calculation, in turn, is no longer to compute answers but to understand and verify the logic of an argument, restate and explain information, and reveal the unstated data.

Embedded in Frankenstein's proposal are the tensions that underlie social-analytical mathematics. She treats the basic materials of math, both in numerical and technical terms, as having already been provided by another source, one that is essentially suspect. In fact, she relies on an unnamed and undescribed form of training that is supposed to equip students with the basic technical skills that enable them to analyze data, which is also, more often than not, gathered elsewhere. This outsourcing of basic technical training is endemic to critical math education.

Frankenstein's approach can partly be described as commercial-administrative mathematics, providing both the methods and raw materials of analysis. This does not, however, cover all grounds. Much of what critical math tries to analyze in fact comes from social-analytical math itself, from statistical or economic analysis performed on social phenomena. Critical math's justified suspicion toward its own raw materials is the result of the observation that social-analytic math can easily turn back into its antithesis, becoming an instrument of commerce and administration rather than a critical tool. Statistics on racial inequality in educational achievement, as Gould (1981), for example, has demonstrated, can be used to critique the education system or to uphold both the existing definition of achievement and its related racial injustices. Within math education the threat is quite subtle, and it is very often ignored by proponents of social-analytical mathematics.

Consider an example from Gutstein (2006, p. 247), who provides his students with the price tags for a B-2 bomber and a college education and then asks them the following question:

> Last June, about 250 students graduated from Simón Bolivar high school. Could the cost of one B-2 bomber give those graduates a free ride to the [University of Wisconsin–Madison] for four years?

On the surface, the question is a critique of policy that allocates money to bombers instead of education. At its depth, however, it reproduces the logic that reduces all objects and decisions to their exchange value. In one stroke of the pen, the B-2 bomber and a college education are reduced to their comparative monetary value. One turns into the other, like a large bill into change. In completing the problem, students do not ask where the bomber comes from, what purpose it serves, or what its dissolution might mean; nor do they ask why a college education in the United States costs as much as it does. If an arms industry lobbyist points out that the construction of the bomber provides jobs for so many workers, and when sold to Saudi Arabia provides so much revenue, and in supporting American interest it upholds the strength of U.S. trade relations, the same monetary logic ends up supporting the necessity of the bomber. In this sense, the well-meaning question still trades in the logic that it purports to attack.

The logic of exchange value cannot easily be subverted by its own tools. Frankenstein's (2009) reframing of the purpose of numbers and calculation is a practical framework against these types of mistakes. But it does not go far enough because it does not contain a theory for critiquing its own tools.

School Mathematics—A Paradigm

All categories of mathematics are formed in relation to the institutional setting in which they are practiced; and schools, as institutions, impose their own conditions. A high degree of scholasticization generally tends to separate aspects of knowledge from direct use in practice and reshapes them as signs of mastery or enculturation. This can apply just as much to a commercial-administrative settings, like the Babylonian scribal school, as it can to philosophical ones, where, as in Medieval European universities, math became a sign of initiation into classical literature (Schrader, 1967) rather than a philosophical pursuit per se.

This tendency has reached its apotheosis in modern public education, where even the least explicit links to practice have disappeared. The false economy of cultural capital sought in grades and certifications transmutes math as it does every other school subject. Children in schools learn what they learn in great part in order to satisfy school requirements, to gain certification, or to garner the approval of teachers. Some theorists of education suggest that school math no longer has any relationship with labor at all (Dowling, 1998), that it is a self-referential discipline born out of the special properties of schooling (Lave, 1992), and the only reason to learn school math is to be able to do more of it later (Lundin, 2010a, 2010b).

Such arguments regarding the separation between labor and learning in schools, powerful as they are, ignore an essential aspect of what constitutes the school curriculum. Schools do not only teach *know-how;* they also teach *attitudes* toward the world, and toward labor in particular (Anyon, 1980). The predominance of a commercial-administrative attitude in elementary education speaks to a particular mind-set. Implicit in using the dominant model of mathematics is the overuse of the intellectual muscles associated with commerce and administration. This type of math is the carrier of the mercantile and administrative relation to life, the tendency to reduce real-world relationships to economic exchanges. School mathematics reflects, and in turn generates, the calculating attitude that results from immersion in the relations of a money economy. The alienation inherent in commercial-administrative math becomes particularly severe when its real function is masked—for example, in story problems where money is replaced with apples and oranges.

The mathematics that dominates elementary education is a diminished version of the math taught in sixteenth-century reckoning schools, having lost the original emphasis on anything other than the most basic commercial situations. Though it reflects the attitude embodied in reckoning, it is geared toward a different purpose than training merchants or even accountants. Elementary math today could better be described as *consumer mathematics.* This downgrade (from merchant/administrator to consumer) is a function of the social downgrade in the role of schools: from schools for the upper-middle classes to schools for the lower classes and then to schools for the population at large. It was not—and still is not—conceivable that the population at large might need to learn actual financial mathematics for the administration of capital and labor. School math, therefore, has been gradually and deliberately reduced to the most basic aspects of reckoning—just enough for shopping or for working as a petty bureaucrat, a soldier, or a cashier. In the process, it has lost whatever social power it possessed. Remnants of philosophical math (in the form of calculus, for example) have suffered a similar sea change as they are cut off from their more expansive purpose in science. How many calculus students know that what they are learning was designed to accommodate physics?

Of course, today's teachers are not reckonmasters. In addition to commercial-administrative math, strong traces of other types of mathematics are evident in the classroom, particularly in private, alternative, and elite schools. It would be impossible to critique every one of these approaches. One thing, however, is clear: none can be complete in itself because at this point in history it is impossible to chart a direct link between education and action, between education and purpose. As long as they are deployed in the isolation of the classroom, these approaches can primarily be analyzed in terms of the social attitudes they promote—terms that are strongly ideological. Therefore, a critique of these approaches is a critique of ideology.

Conclusions and Implications

If we accept that mathematics education, down to its most elementary aspects, is a historical process reflecting economic values and political attitudes, then the implications for theory and practice are enormous. The theoretical framework I outline here can be loosely summarized in four tenets:

1. The economic purpose of math defines its most basic characteristics.
2. The economic characteristics of math impact how it can be taught.
3. The institutional setting within which math is taught also modifies the character of its practice.
4. All of the foregoing aspects impact one another in relation to the socioeconomic forces that shape them.

This framework addresses all those involved in math education—teachers, parents, theorists, proponents of deschooling, and, perhaps most intimately, students. It presents a challenge to individuals and communities to define their own view of mathematics and to not take the discipline for granted at any level. My work in this field indicates various points where accepting such a challenge would lead to new directions in research and practice.

First, this framework challenges the idealistic discourse that underlies nearly all discussions of school mathematics. Rhetoric that presents math learning as an absolute good, as necessary to work and citizenship, masks a deeper discussion regarding the role of labor and politics in society. One cannot take any ideas regarding the "usefulness" of math education for granted. What is needed instead is a precise, dialectical approach that clarifies what shapes curriculum, for whom, and to what end.

Second, this framework challenges the notion that there is a single "basic" math that constitutes the foundation of all other mathematical practices. On the contrary, an educator's worldview and place in society defines how she conceives of such concepts as numbers, precision, and context. While further study may better clarify the connections between worldview and math education, for now it is sufficient to observe that all examinations, without exception, do not test students' "basic knowledge of math." They impose specific notions of what is and is not valid knowledge.

Third, teacher education can no longer only concern itself with *what* teachers teach and *how* they teach it. Any program that views teachers as more than mere functionaries will have to involve an exploration of the *why* questions of mathematics, with the understanding that such an exploration may lead teachers to rebel against the confines and assumptions of their own position.

Finally, this framework provides a critical basis from which we can engage the psychological questions of math education. The statement attributed to Foucault (in Dreyfus & Rabinow, 1982, p. 187) holds true in regard to math education: "People know what they do; frequently they know why they do it; but what they don't know is what what they do does." Nonetheless, any impact that math might have on individuals and society depends on two processes: the one that forms the content of curriculum and the one that delivers it to people. Once these structures are more transparent—and only then—we can finally begin to discuss what math education is making of us as human beings.

Notes

1. A decade before the New Common Core, the extremely similar standards proposed by the National Council of Teachers of Mathematics (2000) exhibited the same willingness to pose unjustified benchmarks for learning. Two decades earlier, in England, the same tendency was evident in the Mathematics Counts reports (Harouni, 2013).
2. The vast majority of American elementary students think of the equal sign as a command to calculate rather than as stating a relationship of equality (Li, Ding, Capraro, & Capraro, 2008).
3. Modern education, like the economic and political systems that support it, is essentially a European product. By shedding light on what was European, we might even achieve the happy side effect of emboldening what was not.
4. "Latin schools" in Prussia and the Netherlands and "schools of the teaching orders" in France and Italy (Jackson, 1906).
5. Davydov's method (Schmittau, 2010), partially reanimated in the United States as the Measure Up curriculum, is a much more thoughtful and theoretically sound artisanal approach. But for that very reason, a discussion of its implications is beyond the scope of this essay.
6. I base my analysis on critical readings of Desrosières (1998) and Porter (1986), among others.

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